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October 29, 1979

DEPARTMENT OF MATHEMATICS

Rm. 2-339

Professor Alan Schoen
Department of Design
Southern Illinois University
Carbondale, Illinois 62901

Dear Professor Schoen:

Professor Rota gave me a copy of your interesting manuscript on rhombic rosettes, and although I have not succeeded in proving your conjectures, I have found a very simple method of constructing symmetric rhombic rosettes. It seems to me unlikely that this method is really new, but I found no references to rhombic rosettes in the literature other than a very brief mention on p. 28 of Coxeter's Regular Polytopes.

The basic idea is to represent a rhombic rosette by an "arrangement of lines." Each rhomb in a rosette is the intersection of the two ladders containing it, and its orientation is determined by these two ladders. Thus to describe a rosette we need only describe the order in which the ladders intersect. We can do this by drawing a picture of n lines, representing the ladders, every two meeting at a point, representing their common rhomb. (See fig. 1.) We have no guarantee that this picture can always be drawn with straight lines, but in "most" cases it can. Thus to construct a rosette we first draw an arrangement of lines with every pair meeting at one point, and no three at a point, and using this as a guide, we construct the corresponding rosette. (See figure 1.) It is clear that if the arrangement of lines is symmetric, then the rosette will be also, and symmetric arrangements are easy to construct. (See figure 2.)

It is natural to ask if arrangements of lines have been, or can be, classified. The only study of arrangements I know of is Branko Grünbaum's Arrangements and Spreads, No. 10 in the Regional Conference Series in Mathematics of the Conference Board of the Mathematical Sciences, published by the American Mathematical Society, Providence, 1972. Grünbaum studies arrangements of lines in the projective plane, which he says are a little simpler than arrangements in the Euclidean plane, though the two are closely related. It seems that a simple classification of arrangements of lines (or arrangements of "pseudolines" -- lines that need not be straight) does not exist, so it is unlikely that a simple classification of symmetric rhombic rosettes exists.

The arrangements that correspond to rosettes are "simple" arrangements -- parallel lines and points meeting three lines are not allowed. However more general arrangements correspond to "generalized rosettes": a pair of parallel lines corresponds to a "double" side (see figure 3) and a point at which k lines meet corresponds to a 2k-gon instead of a rhombus.

I have given a copy of your manuscript to a colleague who knows more about these things than I; he may be able to shed some more light on the subject.

Yours sincerely,

Ira Gessel

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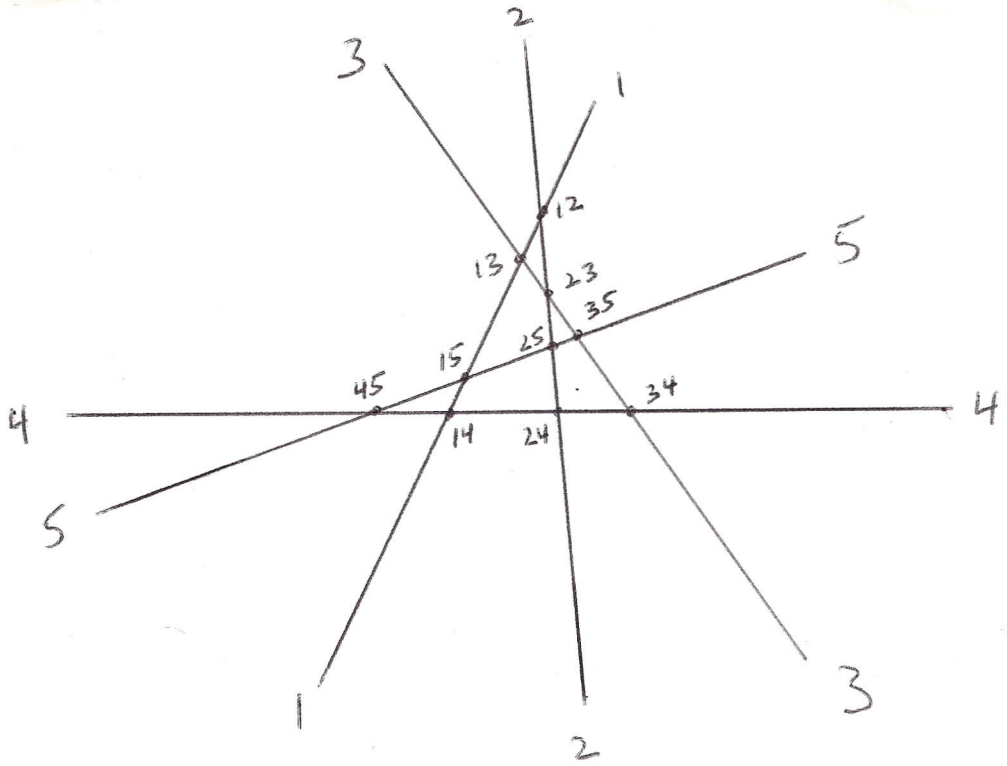
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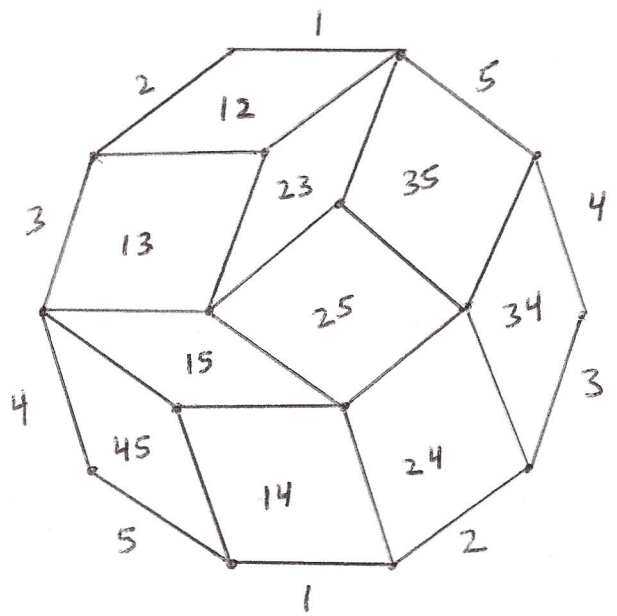
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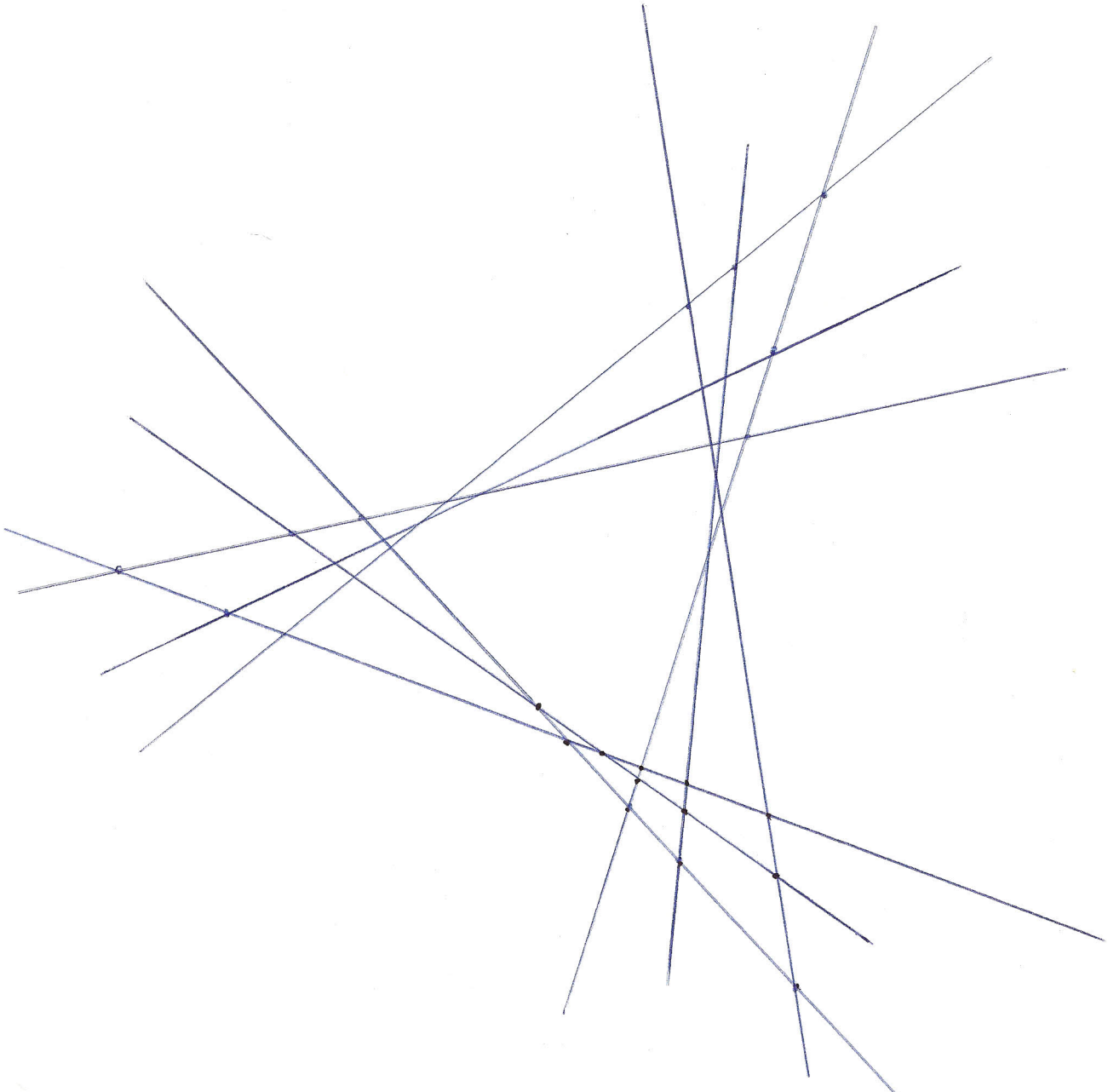
Figure 1

Figure 2



The rosette corresponding to this arrangement is the one pictured on page 2 of your manuscript.

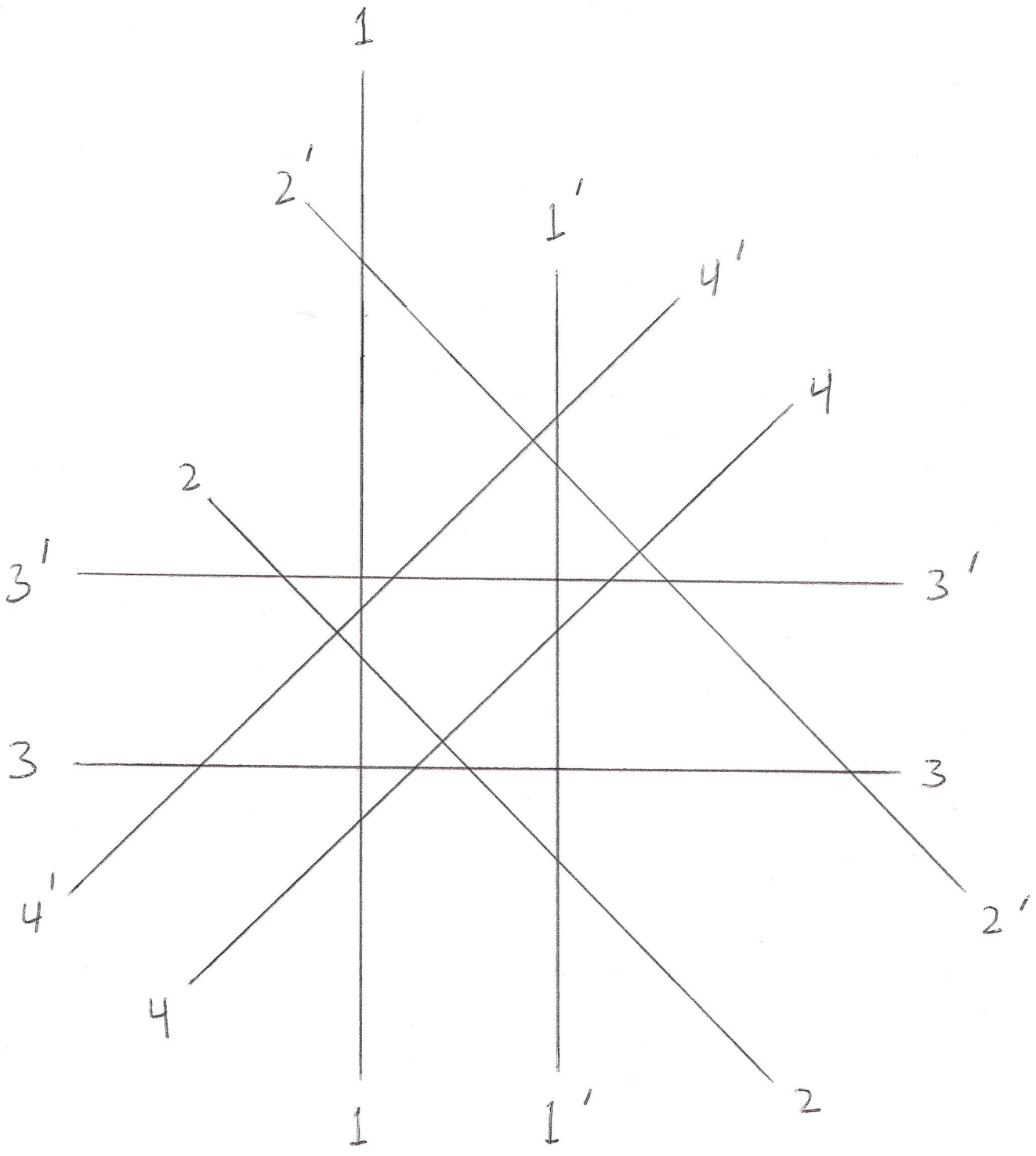


Figure 3a

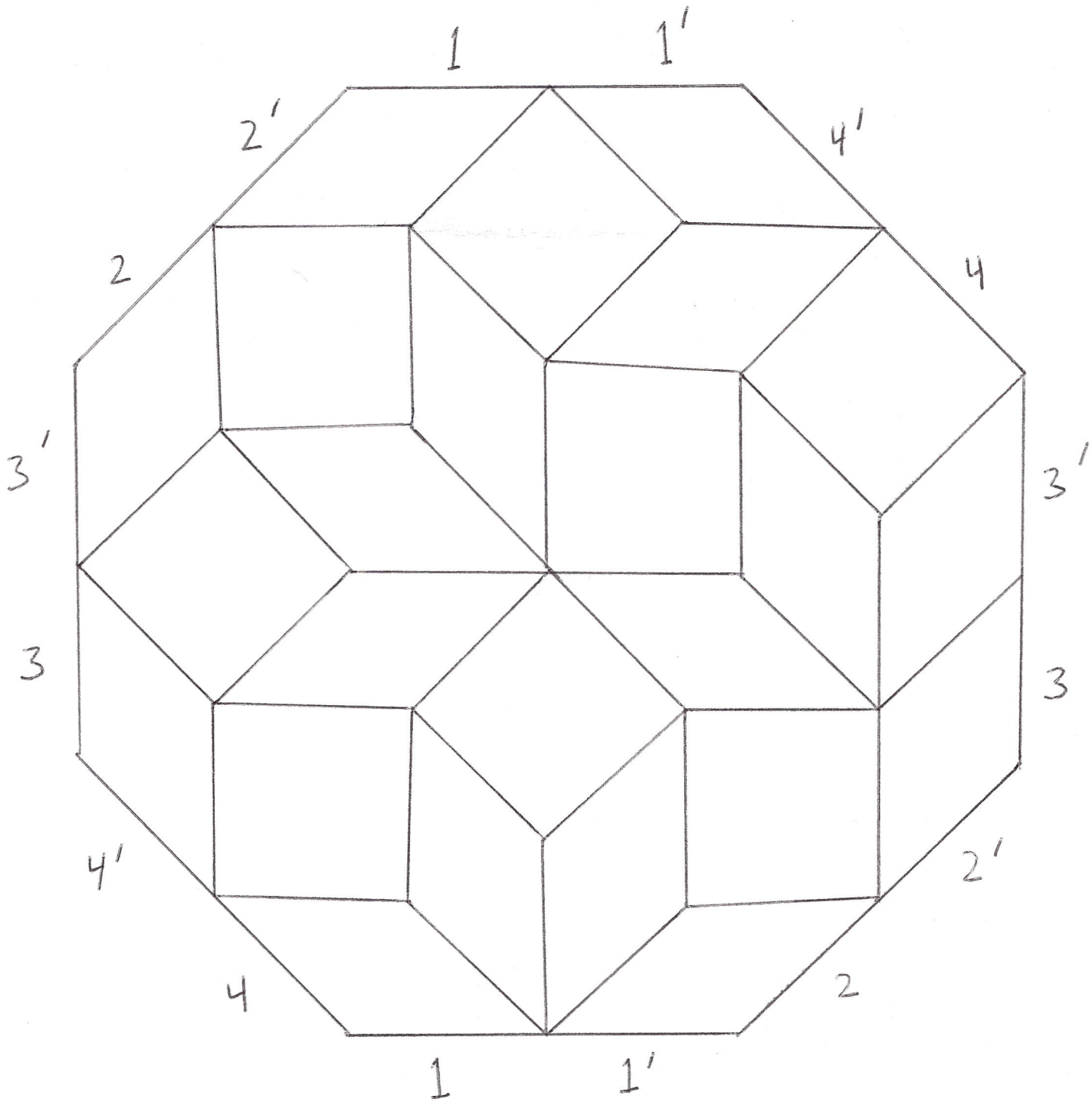


Figure 3b

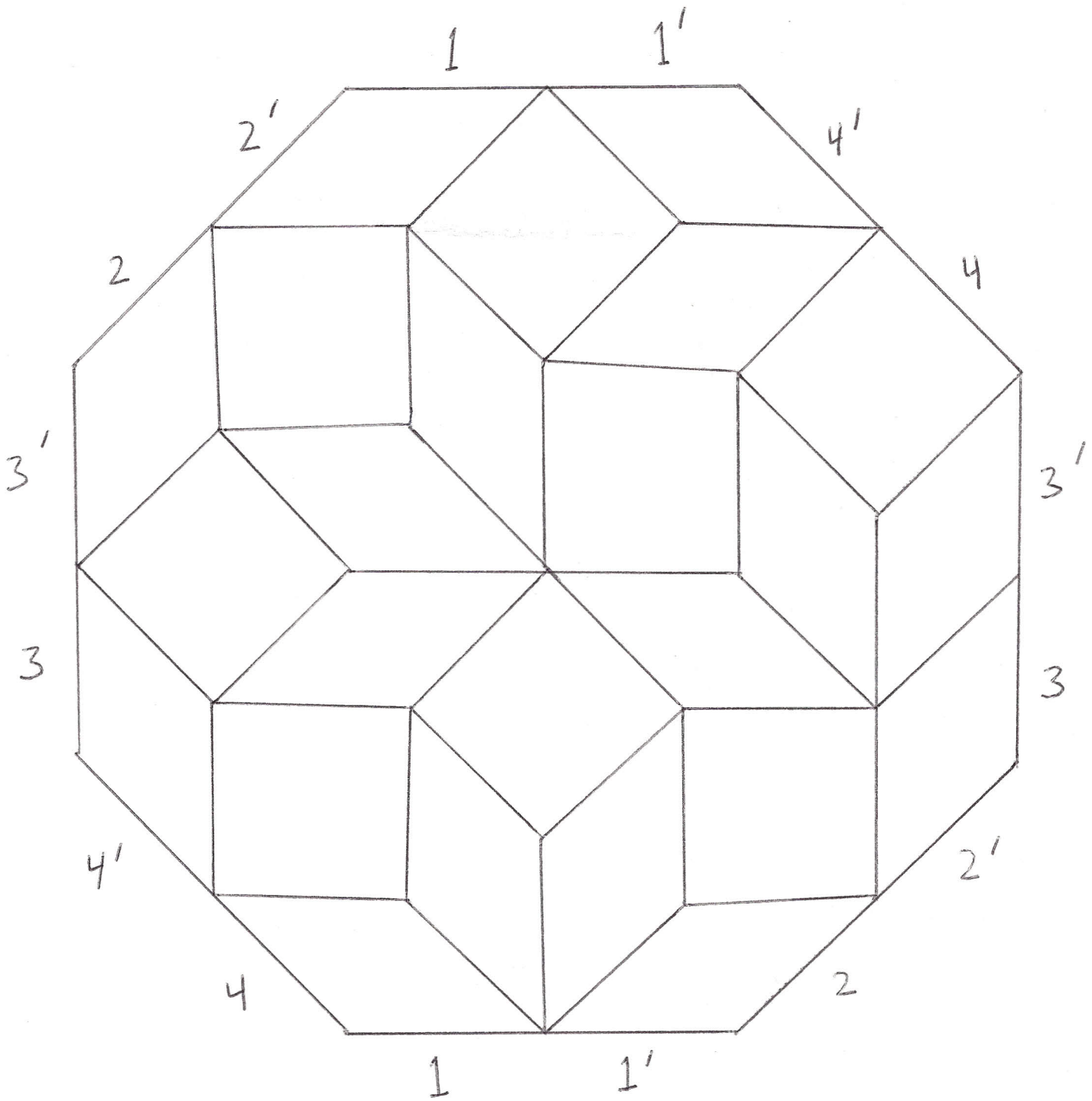


Figure 3b